(1.)

x = fluktuierende Input

$$\boxed{\dot{y} = c(y_g - y) - d\,x\,y} = c\,y_g - (c + d\,x)\,y \qquad (1)$$

$$\dot{y} = 0 \Rightarrow d\bar{x}\,\bar{y} = c(y_g - \bar{y}) = c\,y_g - c\,\bar{y}$$

$$\Rightarrow d\bar{x}\,\bar{y} + c\,\bar{y} = c\,y_g$$

$$\Rightarrow (d\bar{x} + c)\,\bar{y} = c\,y_g$$

$$\Rightarrow \boxed{\bar{y} = y_g \cdot \frac{c}{c + d\bar{x}}} \qquad (2)$$

$$y = \bar{y} + \Delta y \Rightarrow \dot{y} = \dot{\Delta y} \overset{(1)}{=} c\,y_g - (c + d\,x)(\bar{y} + \Delta y)$$

$$\Rightarrow \dot{\Delta y} = \underbrace{c\,y_g - \underbrace{(c + d\,x)\,\bar{y}}_{c\,y_g}}_{0} - (c + d\,x)\,\Delta y$$

$$\Rightarrow \boxed{\dot{\Delta y} = -(c + d\,x(t))\,\Delta y} \qquad (3)$$

Potentialtopf mit fluktuierende Steifigkeit !

$$K_{eff}(t) = c + d\,x(t)$$

$$x = \bar{x} + \Delta x$$

$$\Rightarrow \dot{\Delta y} = [-(c + d\bar{x})\,\Delta y] - d\,\Delta x\,\Delta y$$

μ = Stärke des weißen Rauschens

$$\Delta x = \mu\,\Omega(t) \quad \text{mit} \quad \langle \Omega(0)\,\Omega(t) \rangle = \delta(t)$$

$$\mu = \sigma_x \sqrt{2\tau_x}$$

$$\Rightarrow \boxed{\dot{\Delta y} = \underbrace{[-(c + d\bar{x})\,\Delta y]}_{f(\Delta y)} + \underbrace{[-d\,\mu\,\Delta y]}_{g(\Delta y)}\,\Omega(t)} \qquad (4)$$

2

$$g^2 = d^2\mu^2 \Delta y^2 \quad , \quad \frac{\partial}{\partial x} g^2 = 2 d^2\mu^2 \Delta y , \quad \frac{1}{4}\frac{d}{dx} g^2 = \frac{d^2\mu^2}{2} \Delta y$$

$$\Rightarrow A(s) = -(c + d\bar{x})\, s + \frac{d^2\mu^2}{2} s =$$

$$= -\left(c + d\bar{x} - \frac{d^2\mu^2}{2}\right) s$$

$$B(s) = \frac{d^2\mu^2}{2} s^2$$

$$\Rightarrow \frac{A(s)}{B(s)} = -\left[\frac{c + d\bar{x} - \frac{d^2\mu^2}{2}}{\frac{d^2\mu^2}{2}}\right] \frac{1}{s}$$

$$= -\left[\frac{2}{d^2\mu^2}(c + d\bar{x}) - 1\right] \frac{1}{s}$$

$$\boxed{\frac{A(s)}{B(s)} = -\left[\frac{2c + 2d\bar{x}}{d^2\mu^2} - 1\right] \cdot \frac{1}{s}}$$

keine Näherung $s \to \infty$ erforderlich !!

$$\Rightarrow P(x) \sim \frac{1}{B(x)} \exp\left[\int_{x_0}^{x} \frac{A(s)}{B(s)} ds\right] \sim \frac{1}{x^2} \underbrace{e^{-[\,]\cdot \ln(x)}}_{x^{-[\,]}} \sim x^{-2-[\,]}$$

$$P(x) \sim x^{-2+1-\left(\frac{2c+2d\bar{x}}{d^2\mu^2}\right)}$$

$$\boxed{P(x) \sim x^{-1-\left(\frac{2c+2d\bar{x}}{d^2\mu^2}\right)}} \quad (5)$$

Beachte: Immer PL-Verteil., unabhängig von den Parametern !!

Somit: $$\boxed{\begin{aligned} P(x) &\sim x^{-\alpha} \\ \alpha &= 1 + \frac{2c + 2d\bar{x}}{d^2\mu^2} \end{aligned}}$$