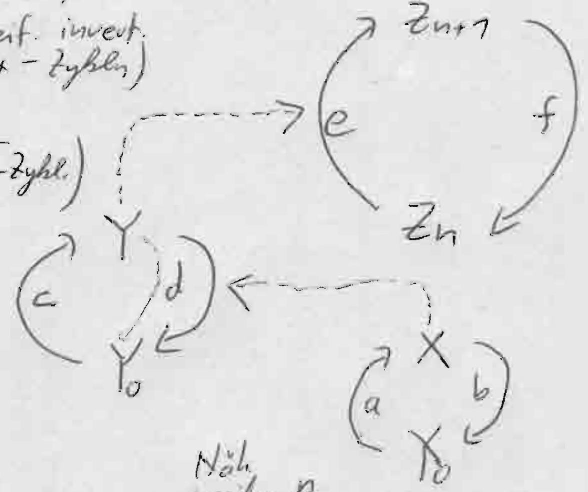
$\dot{n} = e y - f$ (Levy-Faserwachstum)

$\dot{y} = c(y_g - y) - d x y$ (vereif. invert. Marx-Zyklen)

$\dot{x} = a(x_g - x) - b x$ (Prod.-Vern.-Zykl.)



Näh weißes R.

$$x(t) = \bar{x} + \Delta x(t)$$

$$\langle \Delta x(0)\, \Delta x(t) \rangle = \sigma_x^2 e^{-|t/\tau_x|} \approx \underbrace{2\sigma_x^2 \tau_x}_{\lambda} \cdot \delta(t)$$

$$\Big\langle \underbrace{\frac{\Delta x(0)}{\sqrt{\lambda}}}_{\Omega(0)} \underbrace{\frac{\Delta x(t)}{\sqrt{\lambda}}}_{\Omega(t)} \Big\rangle = \delta(t) \quad \Rightarrow \quad \Delta x(t) = \sqrt{\lambda} \cdot \Omega(t) = \sigma_x \sqrt{2\tau_x}\, \Omega(t)$$

$$\dot{y} = \underbrace{\left[c(y_g - y) - d\bar{x} y\right]}_{f(y)} - \underbrace{\left[d \Delta x(t)\, y\right]}_{} - \underbrace{\left[d \sigma_x \sqrt{2\tau_x}\, y\right]}_{g(y)} \Omega(t)$$

$P(x) = $ Gauß: $\quad \bar{x} = x_g \cdot \frac{1}{1 + (b/a)} \qquad \sigma_x^2 = x_g \cdot \frac{ab}{(a+b)^2}$

$\langle \Delta x(0)\, \Delta x(t) \rangle = 2\sigma_x^2 \tau_x\, \delta(t) \qquad \tau_x = \frac{1}{a+b}$

$$P(y) \sim y^{1 - \left(\frac{c + d\bar{x}}{d^2 \sigma_x^2 \tau_x}\right)}$$

$$\langle \Delta y(0)\, \Delta y(t) \rangle = \sigma_y^2 \cdot e^{-t/\tau_y} \qquad \tau_y = \frac{1}{c + d\bar{x}}$$

$$\bar{y} = y_g \left(\frac{c}{c + d\bar{x}} \right)$$

$$g^2 = d^2 \sigma_x^2 2 \tau_x y^2$$

$$B = \frac{1}{2} g^2 = d^2 \sigma_x^2 y^2 \tau_x$$

$$A = c y_g - (c + d\bar{x}) y + \frac{1}{4} \cancel{\frac{d}{dy}} 4 d^2 \sigma_x^2 \tau_x y$$

$$= [c y_g] - (c + d\bar{x} - d^2 \sigma_x^2 \tau_x) y$$

$$\frac{A(s)}{B(s)} = \frac{c y_g - (c + d\bar{x} - d^2 \sigma_x^2 \tau_x) s}{d^2 \sigma_x^2 \tau_x s^2} \xrightarrow[s \text{ groß}]{} -\frac{1}{s} \frac{c + d\bar{x} - d^2 \sigma_x^2 \tau_x}{d^2 \sigma_x^2 \tau_x}$$

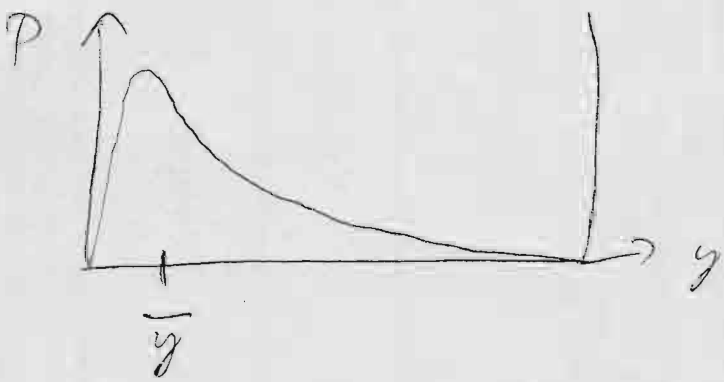
$$= -\frac{1}{s} \cdot \left[\left(\frac{c + d\bar{x}}{d^2 \sigma_x^2 \tau_x} \right) - 1 \right]$$

$$= \overbrace{\left[1 - \left(\frac{c + d\bar{x}}{d^2 \sigma_x^2 \tau_x} \right) \right]}^{\varepsilon} \frac{1}{s}$$

$$\exp\left[\int_{y_{min}}^{y} \frac{A(s)}{B(s)} ds \right]$$

$$\sim \exp\left[\varepsilon \ln\left(\frac{y}{y_{min}} \right) \right] \sim \left(\frac{y}{y_{min}} \right)^{\varepsilon}$$

$$\Rightarrow P(y) \sim y^{1 - \left(\frac{c + d\bar{x}}{d^2 \sigma_x^2 \tau_x} \right)}$$



Bedingungen:

1) $\tau_x \ll \tau_y \iff \frac{1}{a+b} \ll \frac{1}{c+d\bar{x}} \iff a+b \gg c+d\bar{x}$

2) $\bar{y} \ll y_g \iff d\bar{x} \gg c$

$\tau_x = \frac{1}{2}$

Wahl: $a = b = 1 \implies \bar{x} = 50$

$x_g = 100$

$\sigma_x^2 = 100 \cdot \frac{1}{4} = 25$

$\sigma_x = 5.$

Bed. 1) $\frac{1}{2} \ll \frac{1}{c+d\bar{x}} = \frac{1}{c+d \cdot 50} = \frac{1}{10^{-4}+0.05} \approx 20$

$c = 10^{-4}$
$d = 10^{-3}$

Bed 2) $10^{-3} \cdot 50 \gg 10^{-4}$

$\cancel{10^{-2}}$
0.5

$\varepsilon = 1 - \frac{10^{-4} + 0.5}{10^{-6} \cdot 25 \cdot \frac{1}{2}}$

$(1+10)^2 = 3.5 \cdot 10 \cdot d$

$121 = 35 \cdot d$

0.4

Bedingungen:

(2)

1) $\tau_x \ll \tau_y \iff \frac{1}{a+b} \ll \frac{1}{c+d\bar{x}}$

$\iff a+b \gg c+d\bar{x}$

2) $\bar{y} \ll y_g \iff y_g \left(\frac{c}{c+d\bar{x}}\right) \ll y_g$ aus $\dot{y} \overset{!}{=} 0$

$\cancel{\frac{c}{c+d\bar{x}} \ll 1}$

$\cancel{c \ll c + d\bar{x}}$

$\cancel{d\bar{x} \gg c}$

$\cancel{1 \ll 1 + \frac{d\bar{x}}{c}}$

Somit muss $d\bar{x} \gg c$ gelten

3) PL-Exponent $\varepsilon \overset{!}{\simeq} -2{,}5$

$$\varepsilon = 1 - \underbrace{\frac{c+d\bar{x}}{d^2 \sigma_x^2 \tau_x}}_{=:r}$$

$$\boxed{r = \frac{c + d\,(x_g\, ab)/(a+b)}{d^2\, x_g\, \frac{ab}{(a+b)^2} \cdot \frac{1}{a+b}}}$$

$\downarrow$ Bed. (2)

$$\boxed{r \approx \frac{d\bar{x}}{d^2 \sigma_x^2 \tau_x} = \frac{(a+b)^2}{d\,b}}$$

4) $\bar{x} > 5$

Aus (2) folgt:

$$(1): \quad a+b \gg d\bar{x} \quad ; \quad \bar{x} = \frac{x_g\, a}{a+b}$$

$$a+b \gg \frac{d\, x_g\, a}{a+b}$$

$$\frac{(a+b)^2}{a} \gg d\, x_g \qquad (1)'$$

$$\Big\downarrow \ \ \cancel{(3)}: \ r\,d\,b = (a+b)^2$$

$$\frac{r\,d\,b}{a} \gg d\, x_g$$

$$\frac{r\,b}{a} \gg x_g \qquad (1)''$$

$$\Big\downarrow \ \ \cancel{(3)}$$

$$\frac{(1-\varepsilon)\,b}{a} \gg x_g \quad \Rightarrow \quad a \neq b$$